

THE VALIDITY OF THE LAW OF RATIONAL INDICES,
AND THE ANALOGY BETWEEN THE FUNDA-
MENTAL LAWS OF CHEMISTRY AND
CRYSTALLOGRAPHY.

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Some fundamental law of nature governs the position of the faces of a crystal and limits in number the faces which occur on the crystals of any one substance. Crystal faces are designated by intercepts on coördinate axes, which are chosen so as to yield simple relations. Now it is found that the intercepts of the various crystal faces of a given substance, on each coördinate axis taken separately,

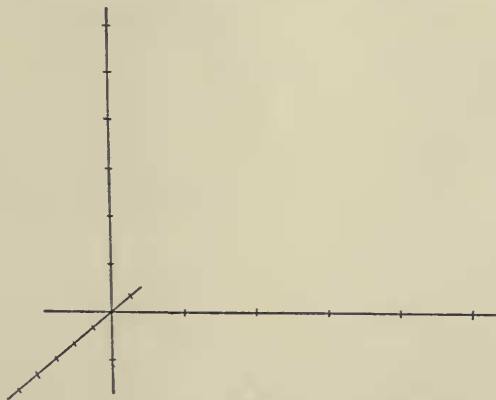


FIG. 1. The coördinate axes of a crystal.

usually bear a simple ratio to each other such as $1:\infty$, $1:2$, $1:3$, $2:3$, $3:1$, etc. A selected face chosen because of its prominence is taken as a standard and the other faces are expressed in terms of it. The selected face is called the unit face, as its intercepts on the three axes establish a unit which, in general, is different for each axis, as represented in Fig. 1. The intercepts of the unit face which are, in

general, irrational constitute the axial ratios which are constants for each crystallized substance. For convenience in calculation the reciprocal ratios of the intercepts are used. These reciprocals are called indices or Miller indices, as Miller, an English crystallographer, was the first to make extensive use of this method. The indices are usually simple numbers such as (110), (210), (130), (211), (321), (441), etc., the unit face being (111).

If we examine the statements concerning the rationality of the indices of crystal faces in text-books and treatises we find a difference of opinion as to the exact definition of the law. Some authors insist that the indices are small whole numbers, while others simply state the fact that the indices are whole numbers, usually, but not necessarily small. One crystallographer, Viola,¹ goes so far as to doubt the validity of the law of rational indices. Another investigator, G. H. F. Smith,² believes that the law of simple rational indices is valid except in one particular instance, that of calaverite from Cripple Creek, Colorado. But, as he shows, by assuming several interpenetrant space-lattices it may be valid even in this case.

Thus there are three possibilities to consider: (1) The indices are always small rational numbers. (2) The indices are rational numbers, but not necessarily small. (3) The indices are not always rational and the law has no meaning. This subject is such a fundamental one in both theoretical and practical crystallography that it seems advisable to enquire into the history and status of the law. Such is the object of this paper.

The credit of the discovery of the rationality of the indices is due to Haüy,³ professor of the humanities in the University of Paris, who developed it from his theory of crystal structure based upon cleavage observations. Haüy believed that crystals are composed of minute cleavage fragments which he called *molécules intégrantes*. Primary faces, according to his view, are due to the association of the *molécules* in parallel position, while secondary faces are due to the omission of *molécules* on the exterior of the crystal in step-like

¹ *Zeitschrift für Kristallographie und Mineralogie*, Vol. 34, pp. 353-388 (1901).

² *Mineralogical Magazine*, Vol. 13, p. 122 (1902).

³ "Essai d'une Théorie sur la Structure des Crystaux." Paris, 1784.

arrangement. According to Haüy the omission is usually of one, two or three, rarely of four or five rows of *molécules*. Fig. 2 shows the production of an (110) face in this manner. If the cubes were very minute the (110) face would appear to be smooth. This epoch-making discovery laid the foundation of crystallography as an exact science and entitles Haüy to the title "father of crystallography." With some modification it has been the guiding principle in crystallography since that time and should not be abandoned unless the evidence is clearly against it.

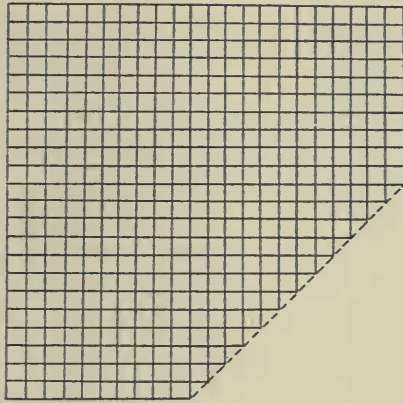


FIG. 2. The production of an (110) face.

Some authors express the fundamental law of crystallography as the law of simple mathematical ratio. Thus Williams⁴ says: "Experience has shown that only those planes occur on any crystal whose axial intercepts are either infinite or small even multiples of unity." Tutton⁵ also says: "The indices of any and every face on a crystal are three small numbers." Small in these quotations is usually interpreted as not more than six. Faces with indices larger than six, according to this view, are accidental and are usually relegated to the list of uncertain forms. There is a tendency to consider forms

⁴"Elements of Crystallography," 3rd ed., p. 26 (1901). Similar statements are also made in the text-books of Bayley, Brush-Penfield, Moses and Parsons, Patton, and Van Horn.

⁵"Crystallography and Practical Crystal Measurement," p. 70, 1911.

with indices at all complex as doubtful even when the measurements indicate the form.⁶

The law of simple mathematical ratio is untenable. There are

LIST OF CRYSTAL FACES WITH COMPLEX INDICES.

	Angle.	Meas.	Calc.	
Octahedrite	1.1.28 \wedge 001	5° 8½'	5° 8'	Robinson, <i>A. J. S.</i> (4), 12, 180.
Baumhauerite...	13.0.12 \wedge 100	48 31	48 31½	Solly, <i>Min. Mag.</i> , 13, 154.
Barite	1.44.0 \wedge 120	40 58	40 57	Brunlechner, <i>Min. Mitth.</i> , 12, 73.
Apophyllite	51.51.50 \wedge 001	61 3	61 1	Ploner, <i>Zs. Kr.</i> , 18, 351.
Fluorite	19.1.1 \wedge 100'	4 13	4 15	Whitlock, <i>Bull.</i> 140, N. Y. St. Mus., 198.
Pyrite	{ 24.15.10 \wedge 111	19 30	19 32	Mauritz, <i>Zs. Kr.</i> , 39, 363.
Tetrahedrite.....	{ 24.15.10 \wedge 100	36 55	36 55	
	{ 35.35.1 \wedge 110	1 9	1 9	Ungemach, <i>Bull. fr. soc. min.</i> , 29, 251.
Tourmaline	20.19.39.1 \wedge 1011	63 0	63 1½	Worobieff, <i>Zs. Kr.</i> , 33, 263.
Vanadinite	43.0.43.40 (ρ)	41 29	41 29	Schaller, <i>Bull.</i> 262, U. S. G. S., 138.
Millerite	31.13.44.0 (ϕ)	13 17	13 17	Palache and Wood, <i>A. J. S.</i> (4), 18, 355.
Pectolite.....	1.0.25 (ρ)	7 29	7 32½	Moses, <i>A. J. S.</i> (4), 12, 98.
Natrolite.....	16.16.17 \wedge 110	64 32	64 33	Zambonini, <i>Zs. Kr.</i> , 34, 581.
Whewellite.....	{ 14.26.1 \wedge 010	32 48½	32 48½	Ježek, <i>Bull. int. l' acad. Sci. Bohême</i> , '09, 9.
	{ 14.26.1 \wedge 101	63 17	63 18	
Iodyrite	33.0.33.2 \wedge 0001	86 20	86 21	Kraus, <i>A. J. S.</i> (4), 27, 218.
Chrysoberyl	11.20.20 \wedge 11.20.20	60 54½	60 54¼	Liffa, <i>Zs. Kr.</i> , 36, 611.
Quartz.....	1.30.29.29 \wedge 0111	1 26	1 25	Gonnard, <i>Zs. Kr.</i> , 29, 323.
Strontianite.....	36.36.1 \wedge 001	1 9	1 9	Beykirch, <i>N. Jb. Min. Beil. Bd.</i> , 13, 423.
Danburite	0.50.1 \wedge 001	87 36	87 37	Weber, <i>Zs. Kr.</i> , 37, 620.
Epidote.....	29.0.1 \wedge 001	63 27	63 27	Zambonini, <i>Zs. Kr.</i> , 37, 13.
Calcite.....	{ 49.41.90.8 \wedge 49.90.41.8	65 29	65 29	Rogers, <i>A. J. S.</i> (4), 12, 43.
	{ 49.41.90.8 \wedge 90.41.49.8	53 49	53 48½	

hundreds of measurements to prove this statement. The accompanying tabulated list gives faces with complex indices for a number of minerals, which list could be greatly extended if space permitted.

⁶ Palache and Wood, *American Journal of Science*, Vol. 18, p. 355, 1904.

These are selected because of the good agreement between the measured and calculated angles. Outside of its position in certain zones the only proof of a face lies in this agreement. Ordinarily an agreement as close as ten to thirty minutes of arc is sufficient to establish a face. For the common form-rich minerals, such as orthoclase, tourmaline, fluorite, magnetite, pyrite, barite, anglesite, calcite, aragonite, cerussite, stibnite, hematite, etc., it is certain that some of the faces have complex indices. To be convinced of this fact let one look over the list of forms of the above mentioned minerals in Goldschmidt's "Krystallographische Winkeltabellen."⁷ For calcite one half of the forms (162 out of 325) have indices greater than 10. The law of simple mathematical ratio is hardly compatible with this fact.

Many crystals have what are called vicinal faces. These are faces with very high indices which replace faces with very simple indices. Thus apparent cubic crystals of fluorite from the north of England are in reality bounded by faces of a tetrahexahedron with the symbol (32·1·0). Here each cube face is replaced by a very low four-faced pyramid. Vicinal faces are often regarded as accidental or in some way irregular and are usually excluded from the law of rational indices as they are of course inconsistent with the law of simple mathematical ratio. As they lie in prominent zones and as their arrangement conforms to the symmetry of the crystal on which they occur, they can hardly be excluded from the list of faces, though their origin is not clearly understood. The only possible argument for excluding them is that the exact indices of such faces can not always be determined, for the agreement between measured and calculated angles must be exceptionally good to establish the face. Miers⁸ found that on alum very flat trisoctahedral faces replace the octahedral faces. In one case the measurements indicated the symbol (251·251·250). As Miers says, this form can not be regarded as established. It may be some other form with a little different

⁷ For recent additions to these lists see Whitlock, *School of Mines Quarterly*, Vol. 31, p. 320; Vol. 32, p. 51 (1910).

⁸ *Philosophical Transactions of the Royal Society, A*, Vol. 202, pp. 459-523 (1903).

symbol. But according to Miers⁹ these vicinal faces on alum probably have rational indices.

If we decide to exclude vicinal faces where shall we draw the line? In the zone of (*hko*) faces, for example, we have a large series of possible faces. (32·1·0) is undoubtedly vicinal and so perhaps are (25·1·0) and (20·1·0). The faces (12·1·0) and (10·1·0) are probably not vicinal, but what of the intermediate faces? Such faces as (12·9·1), (3·5·11) and (11·4·7) can hardly be considered as vicinal, yet they are comparatively complex. There is no exact definition of a vicinal face. As a matter of fact there are all gradations between very simple indices and very complex ones, the limit in complexity apparently being determined by the limits of measurement. There seems to be proof of indices at least as high as 50 (see in the tabulated list apophyllite, danburite and calcite of forms on page 106).

Finding that the indices of crystal faces are often very large numbers a few authors, notably Viola,¹⁰ express the opinion that the law of rational indices has no meaning. For of course if we take the indices large enough any plane can be expressed by whole numbers. It is manifestly impossible to prove by direct measurement that the indices of all crystal faces are rational, for measurements are subject to certain errors, the measured angle rarely ever coinciding with the theoretical angle.

But, as I shall show, *there is indirect proof that the indices of crystal faces are rational numbers.* Since the time of Haüy thousands of crystals have been measured and among all these crystals, which include both minerals and prepared compounds of the laboratory, only axes of 2-fold, 3-fold, 4-fold, and 6-fold symmetry have

⁹ *Philosophical Transactions of the Royal Society, A, Vol. 202, p. 476.*

"One reason why I am led to believe that they are really referable to rational, although not to simple, indices is the following: During the growth of the crystal, one set of vicinal faces is being continually replaced by another along certain zones; . . . but the change is not gradual, neither are the surfaces curved; one plane reflecting surface is replaced by another plane; and although the images may for a time be multiple and confused, sharply defined images emerge successively by the substitution of one image for another *per saltum.*"

¹⁰ *Loc. cit.*, p. 363, "Also kann das allgemeine Gesetz der rationalen Indices keine Bedeutung für die Krystallographie haben."

ever been found. Assuming that these are the only possible symmetry-axes it may be proved¹¹ that crystals consist of regularly arranged particles at small finite distances apart, the arrangement about any particle being the same as about any other. In a regular arrangement of particles of indefinite extent, there is an infinite number of symmetry-axes, some of which are parallel to each other. Let A_1 and A_2 be two parallel symmetry-axes with the minimum distance A_1A_2 between them. A revolution about A_1 brings A_2 to A_3 , and a similar revolution about A_2 brings A_1 to A_4 . By hypothesis the distance A_3A_4 can not be less than A_1A_2 . Therefore the angles of revolution, $A_2A_1A_3$ and $A_1A_2A_4$, can not be less than 60° and therefore no symmetry-axis greater than six is possible. Axes of 2-, 3-, 4-, 5-, and 6-fold symmetry remain to be considered. A revolution of 72° ($\frac{1}{5}$ of 360°) around A_1 and A_2 brings two particles A_3 and A_4 a smaller distance apart than the original minimum distance A_1A_2 . If we take A_3 and A_4 as the original particle a still smaller distance A_5A_6 would result and so on *ad infinitum*. Revolutions of 60° , 90° , 120° , and 180° are not contrary to the hypothesis of a minimum distance. Therefore only axes of 2-, 3-, 4-, and 6-fold symmetry are consistent with a regular molecular structure. While the rationality of the indices may not be subject to direct proof, the symmetry of crystals can be determined by measurement. The fact that only the types of symmetry mentioned have been discovered makes it practically certain that crystals are made up of regularly arranged particles of some kind. Other facts point to the same conclusion.

Assuming homogeneity or regular arrangement of the particles of crystals Barlow¹² has proved that only thirty-two crystal classes or combinations of symmetry elements are possible. It is remarkable that all but one of these classes, viz., the trigonal bipyramidal class (one plane of symmetry and one axis of 3-fold symmetry),

¹¹ Lewis, "A Treatise on Crystallography," pp. 136-137 (1899). Barlow, *Philosophical Magazine* (6th series), Vol. 1, pp. 1-36 (1901).

¹² *Philosophical Magazine* (6th series), Vol. 1, pp. 1-36 (1901). The thirty-two possible crystal classes were also deduced by Hessel in 1830 and independently by Gadolin in 1867. Both of these authors base their work upon the law of rational indices but Barlow's work is based upon homogeneity of structure.

have been found either among minerals or prepared compounds. Moreover, every crystal that has been carefully investigated can be assigned to one of thirty-one out of the thirty-two possible crystal classes.

Physicists in general assume the coarse-grainedness of matter.¹³ It is only fair to assume that in crystals the molecules or particles are arranged in a definite and regular manner. The most comprehensive definition of a crystal is the following: "A crystal is a homogeneous solid, the physical properties of which are the same in parallel directions but, in general, are different in non-parallel directions."

It is necessary to assume a regular arrangement of particles in order to explain many of the physical properties of crystals. On account of the correlation between the geometrical and physical (especially the optical) properties of crystals it is practically certain that the crystal form is an outward expression of a regular internal structure. A regular internal structure accounts at the same time for the constancy of interfacial angles and for diversity in crystal habit. It also seems impossible to explain cleavage in any other way.

If crystals are made up of regularly arranged particles,¹⁴ the centers of which are at small, finite distances apart, all crystal faces necessarily have rational indices for the faces are due to the alignment of particles in parallel position but with the omission of particles in step-like arrangement. A whole number of particles is always omitted.

The indices are not necessarily small numbers but may often be large numbers. Even the highest indices ever assigned to crystal faces such as the vicinal faces of adularia (500·527·0), (250·249·0), and (200·157·0) are simple compared with the number of particles or molecules in a crystal.

¹³Rücker, Report British Association for the Advancement of Science, 1901, p. 12.

¹⁴The size, shape, and nature of the particles are immaterial. It is also immaterial whether they are contact as Haüy believed or widely spaced as modern physicists are inclined to believe. So stripped is the structure-theory of all hypothesis that it becomes a mere geometrical abstraction. It is only necessary to assume that crystals are made up of parts.

The intimate connection between rational indices, molecular structure, and symmetry-axes with periods of 2, 3, 4, and 6 can not be denied. If one is true, it is pretty certain that the others are true. There is direct proof of only one of these facts, viz., symmetry-axes of the kinds mentioned. This is the empirical basis upon which my argument rests. It is absolutely true that only axes of 2-, 3-, 4-, and 6-fold symmetry have ever been found and it is, very probable that these are the only ones that ever will be found. Suppose crystals with an axis of 5-fold symmetry should be included as possible. If five-fold axes are possible, axes of 7-, 8-, 9-, 10-fold, etc., would also be possible, for the minimum possible distance between two particles excludes axes with periods greater than 6 for the same reason that it excludes those with a period of 5. Then instead of 32 crystal classes with one gap to be filled, we should have an indefinite number of crystal classes but with only 31 of them yet found in nature.

Even if we grant that the indices are rational numbers, crystallography would still be very complicated for the number of possible rational ratios is very large. In the orthorhombic system, for example, there are 1,037 possible forms with indices not over 10. Yet for the mineral topaz, which leads all orthorhombic minerals in the number of forms there are only about 125 known forms. For all orthorhombic minerals taken together there are only about 386 known forms with indices not over 10. Of all known substances calcite has the greatest number of crystal forms, about 325 well-established ones with about 140 uncertain ones. Only about a half (162) of the forms have indices greater than 10,¹⁵ yet the possible number of forms in the calcite class with indices not greater than 10 is 876.

We need an explanation that will reconcile the observed fact that the indices are usually simple with the fact that they are occasionally complex, the complexity, in general, increasing with the rarity. Such an explanation is furnished by the structure-theory of Bravais.¹⁶ Bravais assumes that the centers of molecules occupy the points of a space-lattice. Fourteen kinds of space-lattices, con-

¹⁵ That is, h , k , and l in the symbol hkl are not greater than 10.

¹⁶ "Etudes Cristallographiques," Paris (1866).

stituting various styles of crystal architecture, are necessary to account for the crystals of various systems.

The crystal faces of most frequent occurrence are, according to Bravais, those planes in which the points of the space-lattice are most closely packed. These are faces with simple indices as can be seen from Fig. 3. Faces with complex or high indices are planes with the points relatively far apart (Fig. 3). It is a well-known fact

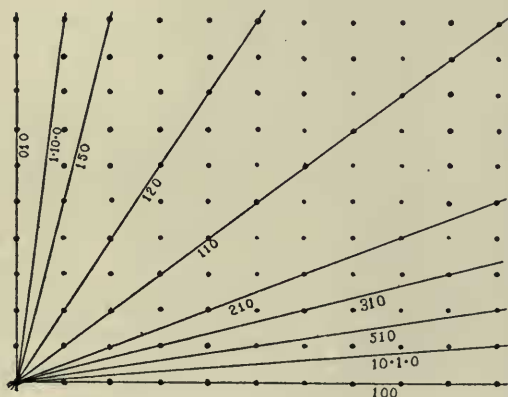


FIG. 3. The relative frequency of occurrence of crystal faces.

that the same forms are not common for every crystal even of the same system. The space-lattice is distinctive for every crystal except those of the isometric system and even in this system there are three kinds of space-lattice possible. The following statistics, compiled from Goldschidt's "Krystallographische Winkeltabellen," will give an idea of the relative abundance of the various forms. Taking the (*hko*) zone for 206 orthorhombic minerals, thus eliminating individual peculiarities, (010) occurs on 134, (100) on 111, (110) on 144, (120) on 66, (210) on 48, (130) on 43, (230) on 29, (320) on 22, (310) on 20, (150) on 17, (430) on 10, (340), (530), and (410) on 9 each, (540) on 8, (610) on 7, (560) and (350) on 6 each, (160), (250), (520), and (10·9·0) on 5 each (650) and (750) on 4 each, (170), (710), and (740) on 3 each, (510) (970), (1·11·0), (1·12·0), on 2 each, and many forms including (7·11·0), (10·7·0), (1·12·0), (16·1·0), (19·20·0), etc., on one each.

Goldschmidt¹⁷ attempts to explain the relative frequency of crystal forms by a different method from that of Bravais. Assuming (100) and (010) as the primary faces in the zone [100:hko:010] secondary faces result by the addition of the indices. Thus adding 100 and 010, index by index, we have as the first complication, 110. Adding 100 and 110, also 110 and 010, we have 210 and 120 as the second complication and so on. The relative frequencies of crystal forms for the *hko* zone are, according to Goldschmidt's law of complication, in the following order.

100										010									
110																			
210										120									
310					320					230					130				
410		520			530		430			340		350			250		140		
510	720	830	730	740	850	750	540	450	570	580	470	370	380	270	150				

Although Goldschmidt's law of complication accounts in a general way for the relative frequency of crystal forms it does not fully explain the observed facts. According to Goldschmidt (210) and (120) should be of equal frequency as should also (310), (320), (230), and (130). Yet (120) occurs on 66 orthorhombic minerals while (210) occurs on only 48. The form (130) occurs on 43 orthorhombic minerals while the other three forms mentioned occur on only 29, 22, and 20 minerals respectively. Out of 206 combinations of anglesite¹⁸ (120) occurs 34 times and (210) only twice. The explanation of these apparent discrepancies is that in the orthorhombic system the *a*-axis is shorter than the *b*-axis and consequently molecules are more closely packed along (120) than along (210). Hence (120) is more frequent than (210). With many orthorhombic crystals, for example cordierite, chalcocite, chrysoberyl, columbite, and witherite (130) occurs to the exclusion of (120).

In the monoclinic system the *a*-axis is either shorter or longer

¹⁷ *Zeitschrift für Krystallographie und Mineralogie*, Vol. 28, pp. 1-35, 414-451 (1897). Abstract by Moses, *School of Mines Quarterly*, Vol. 25, pp. 415-420 (1904).

¹⁸ Hermann, *Zeitschrift für Mineralogie*, Vol. 39, p. 478 (1904).

than the b -axis. Out of 59 monoclinic minerals with a less than unity, (010) occurs on 56, but (100) on only 46 while (120) occurs on 23, but (210) on only 13. Out of 64 monoclinic minerals with a greater than unity, (100) occurs on 60, but (010) on only 43 while (210) occurs on 16, but (120) on only 13. If a is shorter than b , the molecules are more closely packed along (010) and (120) than along (100) and (210), but if a is longer than b the reverse is true.

Out of 168 combinations of orthoclase (including microcline) crystals given in Hintze's "Handbuch der Mineralogie," (010)

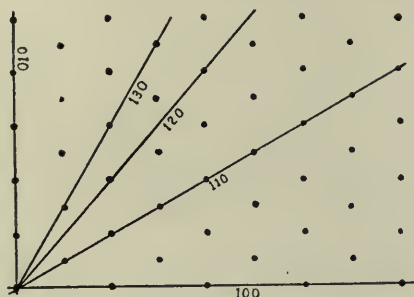


FIG. 4. The probable structure of orthoclase.

occurs on 133, but (100) on only 22. The form (130) occurs 70 times but (120) occurs only once! This remarkable case is explained by assuming the structure to be that of the monoclinic or clinorhombic prism, one of the space-lattices of Bravais. It can be seen from Fig. 4 that the molecules are more closely spaced along (130) than along (120).

It is certainly true that the form-series differs for various crystals, but according to the law of complication the form-series should be alike for all crystals and all systems.¹⁹

For those who are familiar with chemistry, the whole matter of indices, rationality, crystal structure, and relative frequency of crystal faces may be cleared up by considering the analogy between the fundamental laws and theories of chemistry and those of crystallography.

¹⁹ Goldschmidt (*loc. cit.*) explains the differences in the form-series by assuming outer disturbing influences. These undoubtedly have an effect but certain discrepancies are more easily explained by the law of maximum reticulate density.

For the crystals of any one substance the angles between corresponding faces are constant. This law is known as the law of constancy of interfacial angles. It corresponds to the law of definite proportions in chemistry.

The proportions in which two elements combine determines the atomic weight of the elements. In an analogous way the intercepts, which are determined by certain constant interfacial angles, establish the axial ratio which, like the atomic weight, is a constant.

Crystal measurement corresponds to quantitative analysis in chemistry. Exact measurements establish the axial ratio of a crystal just as exact analyses establish the atomic weight of an element.

Two chemical elements *A* and *B* unite not only to form the compound *AB* but also the compounds *AB*₂, *A*₂*B*₃, *A*₃*B*, etc. This fact is known as the law of multiple proportions. These proportions for most chemical compounds are usually simple but in many compounds, especially those containing silicon or carbon, they are often far from simple. Among silicate minerals we have such compounds as $\text{Mg}_5\text{Al}_{12}\text{Si}_2\text{O}_{27}$ and $\text{H}_{20}\text{Mg}_{11}\text{Al}_8\text{Si}_6\text{O}_{45}$. Among organic compounds we have $\text{C}_{60}\text{H}_{122}$, $\text{C}_{17}\text{H}_{23}\text{NO}_3$, $\text{C}_{27}\text{H}_{46}\text{O}_{14}$, and many others with fifty or more carbon atoms in the molecule. In spite of these complex formulæ all chemists accept the law of multiple proportions as an established fact. Without it chemistry would scarcely deserve to be called a science. The law of rational indices in crystallography corresponds to the law of multiple proportions in chemistry. The same difficulties are encountered in crystal measurement as in quantitative analysis. That is, there are certain errors which usually render it impossible to prove absolutely the law of rational indices or the law of multiple proportions.²⁰ According to Jaquet the formula of hemoglobin (of the dog) is $\text{C}_{758}\text{H}_{1203}\text{N}_{195}\text{S}_3\text{FeO}_{218}$. This formula can hardly be regarded as established. It may be a little different but it is very probable that these elements unite in definite proportions. This is exactly analogous to vicinal faces such as (251·250·250) observed on alum by Miers.

The law of multiple proportions was deduced by Dalton from his atomic theory before there were accurate analyses to prove it,

²⁰ Organic chemistry has an advantage over inorganic chemistry in that the formulæ may usually be determined by the method of formation.

just as the law of rational indices was deduced by Haüy from his theory of crystal structure. If chemical compounds are made up of atoms they must necessarily unite in definite proportions. This it will be recalled is precisely analogous to the argument used for proof of the rationality of the indices. If crystals are made up of particles or molecules, the crystal faces necessarily have rational indices.

Two or more given elements do not unite in all possible proportions but in a comparatively few, usually simple, proportions which we explain by the term valence. There are but two oxides of mercury Hg_2O , and HgO which we explain by saying that the valence of mercury is one and two. This is analogous to the limitation imposed by the law of complication of Goldschmidt or the law of maximum reticulate density of Bravais.

To complete the analogy between the laws and theories of crystallography and chemistry let us consider the periodic law and its analogue. Mendeléef, the Russian chemist, predicted the existence of several chemical elements, scandium and gallium, which he called ekaboron and eka-aluminum, before they were discovered. Not less remarkable was the deduction by Hessel, a German mathematician, of the thirty-two possible types of symmetry in crystals, assuming 2-, 3-, 4-, and 6-fold symmetry-axes, in 1830, at a time when only about half of them were known. Of the thirty-two possible types of symmetry, only one remains to be found.

SUMMARY.

Judging from various text-books and articles a difference of opinion exists as to the exact meaning of the law of rational indices. Some authors limit the indices to simple numbers while others admit that occasionally the indices are large numbers. Unfortunately this question can not be decided by direct measurement of the angles on account of errors in measurement. As crystals possess axes of only 2-, 3-, 4-, and 6-fold symmetry they must consist of regularly arranged molecules, or particles of some sort, whatever their nature may be. Crystal faces, then, necessarily have rational indices. The indices are usually small numbers but may also be

complex, the complexity in general increasing with the rarity of the face. The structure theory of Bravais offers a satisfactory explanation of the abundance of faces with simple indices and the rarity of faces with complex indices. There is a remarkable analogy between the fundamental laws of chemistry and crystallography.

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